Controlling the NSGA-II Algorithm convergence toward a fixed Pareto-optimal solution for the Gross Domestic Product quarterly disaggregation

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Outline

Research context

- In recent years, the establishment of quarterly national accounts has become a matter of concern for most developing countries.
- Hence, in the context of the quarterly publication of annual national accounts, several methods and algorithms are used. [refer to IMF 2017 manual on quarterly accounts].
- Moreover, each method used so far in this context must give on solution for the solved problem.
- This research is an extension of previous works in the same field and is motivated by the need to ensure the unicity of the solution of the quarterly disaggregation of Gross Domestic Product (GDP) by controling certain parameters.

Goal of the research

The main goal of the research

The aim of this research is to test the adapted NSGA-II Algorithm robustness and speed of convergence towards single Pareto-optimal solution.

Otherwise, this is about to:

- To perform several tests the algorithm the convergence and the speed of NSGA-II Algorithm by a combinaison of the value of input key parameters such as the population size (pop_size), the iteration number (no_rum), the maximum number of generations (gen_max) and the mutation distribution index in the polynomial mutation $(etam)$;
- To identify respectively their threshold to ensure a single Pareto-Optimal solution in order.

Multiobjective Programming for GDP Disaggregation

MOPTD: Model of GDP Quarterly Disaggregation

$$\min_X \quad F(X) = [f_1(X), f_2(X), \ldots, f_M(X)] \quad (1)$$

s.t

$$X = (X_1,\ X_2, \ldots, X_M) \quad (2)$$

$$X_k = (X_{k,t})\,,\ t = 1,\ 2,\ 3, \ldots, 4T, \quad \forall\ k = 1,\ 2, \ldots, M \quad (3)$$

$$-X_{k,t} \leq 0,\ \ \forall\ t = 1,\ 2,\ 3, \ldots, 4T, \quad \forall\ k \quad (4)$$

$$\sum_{t=4y-3}^{4y} X_{k,t} - Z_{k,y} = 0, \quad \forall\ t = 1,\ 2, \ldots, 4T, \quad \forall\ k \quad (5)$$

$$\sum_{t=4y-3}^{4y} \frac{X_{k,t}}{I_{k,t}} \times \eta_{k,t} - \frac{Z_{k,y}}{\sum_{t=4y-3}^{4y} I_{k,t}} = 0,\ \ \forall\ y = 1,\ 2, \ldots, T,\ \ \forall\ k \quad (6)$$

$$\text{Given}:\ GDP_t = \sum_{k=1}^{M} X_{k,t},\ \forall\ t = 1,\ 2, \ldots, 4T \quad (7)$$

Where components are defined as following

- The objective functions are given by:

$$f_k\left(X_1,\ X_2,\ldots,X_M\right)=\sum_{j=1}^{M}\sum_{t=2}^{4T}\overline{W}_{j,k}\left(\frac{X_{j,t}}{I_{j,t}}-\frac{X_{j,t-1}}{I_{j,t-1}}\right)^2, k=1,\ldots,M \quad (8)$$

- $Z_{k,y}$: is the given value of the branch k accounts for the year y
- $X_{k,t}$: is the anknown the quarterly accounts for branch k at quarter t
- $I_{k,t}$: is the value at quarter t of the indicator related to branch k
- $\overline{W}_{j,k}$: is the interaction of the branch j on the branch k, which is the average of the product j share demanded by branch k, $\overline{W}_{j,k}=1$ if $j=k$ and $0\leq\overline{W}_{j,k}<1$ if $j\neq k$
- The quarterly weights of the indicator are given by:

$$\eta_{k,t}=\frac{I_{k,t}}{\sum\limits_{r=4y-3}^{4y}I_{k,r}},\quad\forall\ k=1,\ 2,\ldots,M;\quad\forall t=1,\ 2,\ldots,4T. \quad (9)$$

We consider the following description of the indexes

- M : is the number of national accounts branches, equal to the number of objective functions
- T : is the number of years for national accounts observation
- $k \in \{1, 2, 3, \ldots, M\}$: generic year index
- $y \in \{1, 2, 3, \ldots, T\}$: generic year index
- $i \in \{1, 2, 3, 4\}$: quarterly index
- $t \in \{1, 2, 3, \ldots, 4T\}$: generic index of quarters on the T years' period of data observation, obtained from i and y using an operator proposed R. A. Essessinou et al. (2019)

Methods of solving the problem

The used approach

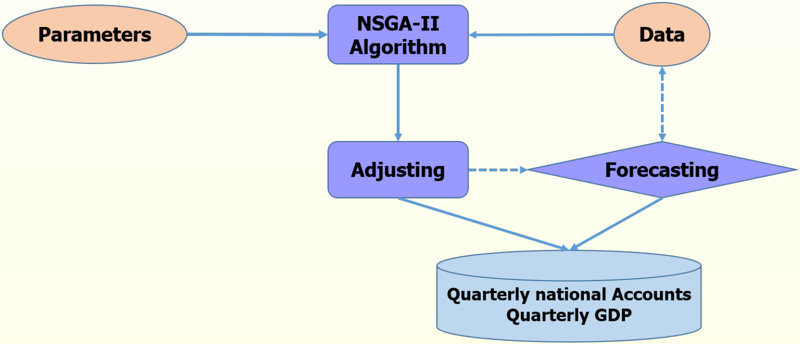
- The optimal Pareto soutions are targeted by using fast elitist non-domination sorting and crowding distance assignment.
- The simulation method for the model solving is based on the NSGA-II algorithm developed in the literature for multiobjective optimization.
- For more information see Kalyanmoy Deb, Amrit Pratap, Sameer Agarwal, and T. Meyarivan, A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II, IEEE TRANSACTIONS ON EVOLUTIONARY COMPUTATION, VOL. 6, No. 2, APRIL 2002.

A brief overview of the NSGA-II

NSGA-II pseudo-code

While (total number of iterations not completed)
Generation of the initial population
Repeat
    While (Population is not classified) do
        Evaluation and Normalization of the constraints
        Search for undominated individuals
        Replacement of individuals
    End while
    Selection
    Crossover
    Polynomial mutation
Until (Criteria for shutdown achieved)
Recombination of optimal Pareto solutions
End while

Architecture



MOPTD Algorithm speudo-code

BEGIN

Step 0: Assigning global parameters

Step 1: Setting the decision space upper bound and lower bound for the target variables

Step 2: Initializing all general parameters

Step 3: Interpolation of accounts with quarterly indicators

Step 4: Initialization of the other NSGA-II input parameters: number of constraints, number of variables, the size of initial population to be generated, the number of iteration, the maximum number of generations

Step 5: Calling the Model function

Step 6: Make a combinaison of the input key parameters: the population size (pop_size), the iteration number (no_rum), the maximum number of generations (gen_max), the mutation distribution index in the polynomial mutation $(etam)$

Step 7: Computing the NSGA-II algorithm

Step 8: Check if the NSGA-II Algorithm converges toward a fixed Pareto-optimal

Step 9: If it is a single Pareto-optimal solution, go to the next step. If not, go back to the step 6

Step 10: Recovering of the optimal solution provided by NSGA-II

Step 11: Adjusting the optimal solution provided by NSGA-II

Step 12: Predicting if necessary the quarterly GDP for next quarters.

END

The adopted strategy

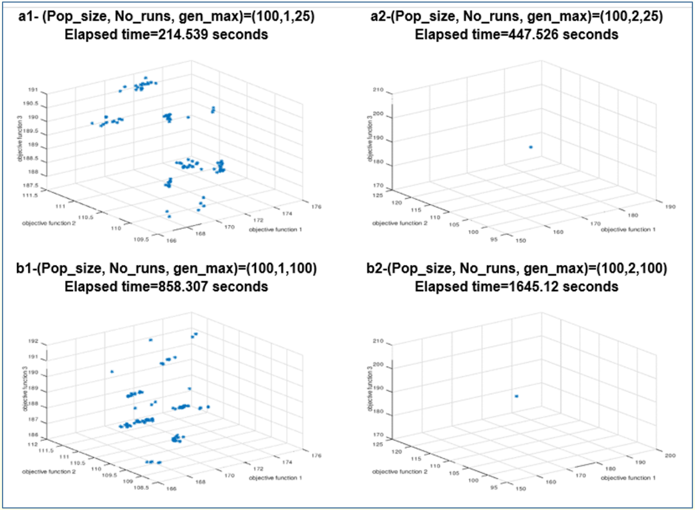
- The data are collected from the National Statistics Office of Benin. The time series of annual national accounts (ANA) for 19 branches are available from 1999 to 2015. Quarterly releted indicators are also available from 1999:Q1 to 2015:Q4.
- Grouping of branches into three sectors with the correspondes branches in national account system: Primary sector, Secondary sector, Tertiary sector.
- The composite quarterly indicator relates to each sector is obtained by a Factorial Analysis on all the indicators soosiated to the correponding branches of the sector.
- Characteristics of the model:

3 objective functions, 204 variables, 306 constraints.

Optimal pareto front

- The algorithm is implemented with OCTAVE on a personal computer under Windows 10 Pro,Processor inte (R) core (IM) i7-6600U CPU@260GH 2.81GHz 16GB RAM.
- Based on the findings in literature, several values were tested for the parameters: pop_size, no_rum, gen_max and $etam$.

Optimal pareto front



The parameters combinaison results

TABLE 1: SENSIBILITY OF MUTATION DISTRIBUTION INDEX ON THE CONVERGENCE AF THE GDP PROGRAM ALGORITHM

(*pop_size, no_rum, gen_max*)	Mutation distribution index (etam)	Running Time (in second)	Number of Pareto-optimal solutions
(10, 1, 25)	0	18.292	10
(10, 1, 25)	1	19.2736	8
(10, 1, 25)	10	19.6863	9
(10, 1, 25)	100	17.6145	9
(10, 1, 25)	500	18.4421	9
(10, 1, 25)	1000	19.0442	8
(10, 1, 25)	2000	17.6917	6
(10, 1, 25)	5000	18.6984	8
(100, 10, 25)	0	1656.67	1
(100, 10, 25)	100	1703.82	1
(100, 10, 25)	200	1733.95	1
(100, 10, 25)	500	1769.96	1
(100, 10, 25)	1000	1751.14	1

Conclusion and next step

- On the test problem, NSGA-II is able to converge closer to a single fixed Pareto-optimal front with the GDP quarterly disaggregation problem.

- In the convergence speed test by controlling both the mutation distribution index and the parameter triplet $(pop\_size, no\_rum, gen\_max)$, it is clear that the mutation distribution index alone doesn‚Äôt influence the speed of algorithm convergence.

- Finally, if the parameter triplet $(pop\_size, no\_rum, gen\_max)$ is judiciously chosen so that $pop\_size \geq 100$, $no\_rum > 1$ and $gen\_max > 2$, NSGA-II algorithm converges to a single Pareto optimal solution located in a fixed cuboid for the test problem for any value of the mutation distribution index.
- The next step in our research consist on set up an algorithm usefull for more than three objective functions.

References

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