Stochastic Optimization for Macroscopic Urban Traffic Model with Microscopic Elements

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Abstract—The Cell Transmission Model for Urban Traffic (CTM-UT) is a macroscopic model with microscopic traffic elements for complex traffic dynamics of arterial and node. In this paper we presented the surrogate method (SM) to optimize the signal synchronization problem considering every turns in accord with the CMT-UT framework. Some extensions to improve the efficiency and efficacy of the surrogate method are proposed. Preliminarily experiments indicate that the CTM-UT can be well-applied for the signal synchronization of real-time urban traffic with good trade-off of efficiency respect to a macroscopic model.

Keywords—urban traffic network; stochastic control; traffic signal optimization; simulation model;

I. INTRODUCTION

In the last years, the development of intelligent transport systems (ITS) has increased the need to define the simulation methods to accurately estimate the travel time prediction but at the same time they must be efficient to be applicability for the optimization of real-time traffic. For example, optimization tools that use multiple traffic simulations have been defined (e.g., TRANSYT-7F and SYNCHRO) and also optimization tools based on hybrid traffic flow models (e.g. [1], [2], [3]) are introcted. These last combine macroscopic (or mesoscopic) model with microscopic model. The choice of the correct optimization method in relation to the characteristics of the traffic network is complicated, studies have not defined exhaustively the reasons behind the inconsistencies between optimization and simulation tool results.

The researchers have attempted to increase the level of detail of macroscopic model. For example, the well-known Cell Transmission Model (CTM by Daganzo [4]-[5]) has been extended with new formulations to model practical applications. Originally, the CTM has been designed to provide a simple representation of traffic on a highway. Through important features such as queue formation, queue dissipation and kinematic waves, the CTM can give a faithful representation of the real dynamics of traffic.

In recent years, has been defined a framework, based on CTM, named Cell Transmission Model for Urban Traffic (CTM-UT). The CTM-UT can mimic microscopic traffic elements for complex traffic dynamics on arterial and node. It is a good trade-off between accuracy and computational complexity, respect to the microscopic model. Previous studies ([10],[11]) have demonstrated the good trade-off of accuracy of the CTM-UT in comparison with microscopic simulation tools (i.e., VISSIM and SUMO). In particular, the CTM-UT is suitable to represent: i) complex flow interactions between neighboring turning movements when entering in the channelized lanes upstream intersection; ii) conflict among crossing flows at complex signalized or unsignalized intersection with the capacity determination for minor flows.

We have adopted the surrogate method ([8]) to optimize the signal setting problem. The SM is based on an on-line and stochastic control scheme. This paper presents reviews of aspects related to solution exploration and gradient estimation.

The aim of this paper is the optimization of signal setting utilizing the CMT-UT to simulate the objective function. In the paper are presented: a general algorithm to maximize crossing flows to an urban intersection respecting the boundaries constraints; the formulation for optimization model to define green splits constraints for every turns in accord with the CMT-UT framework.

Moreover, new extensions are proposed to improve the efficiency and efficacy of the SM application.

II. BACKGROUND FORMULATION

A. CTM-UT

With the CTM-UT the arterial can be distinct in two zones: a downstream queue storage area where vehicles are split into specific lanes dedicated to different turning movements, and an upstream merging zone where the turning movements are mixed. The CTM-UT captures urban traffic dynamics taking into account complex flow interactions among lane groups at upstream of signalized intersections. For the node, the CTM-UT can represent the connection of the demand upstream intersections to the supplies downstream intersection; the demand percentages of turns for
every single lane; merging flows or conflict among crossing flows with the capacity determination for minor flows.

The following formulation defines the flow propagation into the cell \( i \) belong to the links \( a \) of traffic network at period \( k \); \( y_i^{ab} \) is the flow into cell \( i \) of link \( a \) and direct to lane \( b \); \( n_i^{ab} \) is the number of vehicles contained in cell \( i \) of link \( a \) and direct to lane \( b \); \( Y_i^a \) is the maximum number of vehicles that can flow into cell \( i \) of link \( a \); \( w_i^a (F_i^a(k) - n_i^a(k))v_i^a \) represents the total space available in the into cell \( i \) of link \( a \).

### B. Propagation into link

**Upstream arrives**, for \( i = 1 \) the inflow into the first cell of link is calculated by

\[
y_i^a(k) = \min \left\{ \frac{w_i^a(F_i^a(k) - n_i^a(k))}{v_i^a} \right\}
\]

(1)

where \( n_i^{f,a}(k) \) is the flow demand from fictitious node \( f \) at period \( k \).

Inflow of the cells belongs to the **merging zone** of link \( a \) can be described with the following equation

\[
y_i^a(k) = \min \left\{ \frac{n_i^a(k)}{v_i^a} , \frac{Y_i^a}{v_i^a} , 1 \right\}
\]

(2)

and for estimate flow into cell \( i \) of link \( a \) and direct to lane \( b \) we have:

\[
y_i^{ab}(k) = \Phi_{ab} y_i^a(k) , \quad 1 < i < N-I
\]

(3)

When \( i = N-I+1 \), max flow of **downstream channelized zone** can be calculated by

\[
\overline{y}^{ab}(k) = \min \left\{ \frac{\Phi_{ab} n_i^{ab}(k)}{w_i^{ab}(k) + n_i^{ab}(k) - n_i^{ab}(k)} \right\}
\]

(4)

It permits to maximize the demand of upstream lane \( n_i^{ab} \) considering the maximum capacity of lane \( \alpha^{ab} Y_{N-I}^a \) and the necessary restriction to ensure that the inflow \( \overline{y}^{ab} \) doesn’t exceed the available capacity. \( \overline{y}^{ab} \) represents the total space available in the downstream cell \( i \).

Because of conflict between turning vehicles and ahead vehicles, the total inflow of channelized zone can be formulated as follows

\[
y_{N-I+1}^{a}(k) = \min_{b \in B_a} \left\{ \frac{\overline{y}^{ab}(k)}{\alpha^{ab}} \right\}
\]

(5)

Inflow of each direction can be calculated as

\[
y_{N-I+1}^{ab}(k) = \Phi_{ab} y_{N-I+1}^{a}(k)
\]

(6)

### C. Surrogate Method

We utilized the approach proposed by [6] and applied for the first time in traffic problem [7], based on the idea of transforming a *discrete* optimization problem into a “surrogate” *continuous* optimization problem which is not only easier to solve, but also much faster using standard gradient-based approaches.

The Traffic synchronization problem can be formulated as follows:

\[
\min_{r \in A_d} J_d(r)
\]

(10)

where \( r \) is an \( N \)-dimensional decision vector with \( r_n \in Z_+ \) denoting the green time ratio for intersection link \( i \), the capacity constraint is \( A_d \) and \( J_d(r) = E[L_d(r,q)] \) is the objective function, where \( L_d(r,q) \) is the total delay on the network when the state (green split vector) is \( r \) over a specific sample path (link flow) denoted by \( q \), and \( E[] \) denotes the expected value.

First, initialize \( \rho_0 = r_0 \) and perturb \( \rho_0 \) to have all components non-integer. Then, for any iteration \( k = 0,1,… \)
1) Determine \( N(\rho_k) \), the set of all feasible neighboring discrete states of \( \rho_k \):
\[ N(\rho_k) = \{ r | r = \inf \rho_k + \tau \text{ for all } \tau \in \{0, 1\}^N \cap A_k \} \]
2) Determine \( S(\rho_k) \), a selection set to define a set whose a convex hull includes \( \rho_k \) (using [8]).
3) Select a transformation function \( f_k \in F_{\rho_k} \) such that 
   \[ r_k = f_k(\rho_k) = \arg \min_{r \in S(\rho_k)} \| r - \rho_k \|_2 \]
4) Evaluate the gradient estimation \( \nabla L_c(\rho_k) = [\nabla_1 L_c(\rho_k), ..., \nabla_N L_c(\rho_k)]^T \),
   using the following relationship 
   \[ \nabla_j L_c = L_d(\rho_j^*) - L_d(\rho_k^*) \]
   where \( k \) satisfies \( r^j - r^k = e_j \).
5) Update state: \( \rho_{k+1} = \pi_{k+1}(\rho_k - \eta_k \nabla L_c(\rho_k)) \).
6) If some stopping condition is not satisfied, repeat steps for \( k + 1 \). Else set \( \rho^* \).

Finally, we obtain \( r^* \) as one of the neighboring feasible states in the set \( S(\rho^*) \). It is important to notice that to evaluate the gradient estimation in [4], we need to calculate \( n + 1 \) times the value of the objective function, so this estimation is dependent of the decision vector dimension.

The basic idea of this approach is to solve the continuous optimization problem above with standard stochastic approximation methods and establish the fact that when (and if) a solution \( \rho^* \) is obtained it can be into some point 
\[ r = f(\rho^*) \in A_k \] which is in fact the solution of step 4.
Note, however, that the sequence \( \{\rho_k\} \), \( k = 1, 2, ... \) generated by an iterative scheme for solving step 5, consists of real-valued allocations which are infeasible, since the actual system involves only discrete resources. Thus, a key feature of this approach is that at every step \( k \) of the iteration scheme involved in solving step 5, the discrete state is updated through 
\[ r_k = f_k(\rho_k) \] as \( \rho_k \) is updated. This has two advantages: first, the cost of the original system is continuously adjusted (in contrast to an adjustment that would only be possible at the end of the surrogate minimization process); and second, it allows us to make use of information typically employed to obtain cost sensitivities from the actual operating system at every step of the process.

III. GENERAL NODE MODEL

A good intersection model should be suitable to estimate adequate supply and demand values to the upstream links and to links downstream of the intersection, respectively. To model intersections is fundamental to consider their relationship with link boundary conditions, taking into account the behavioral component, the partial supplies and demands of flow. As far as it concerns the node, its space dimension and traffic dynamics are not taken into consideration.

We present a node algorithm for urban traffic to define the inflow and outflow of every intersection \( m \) respecting all boundary constraints. In particular the algorithm calculates the flows disaggregated by turn \( y_{abc} \), lane \( y_{ab} \), intersection outflow \( y_{c} \), intersection inflow \( y_{abc+1} \) with the relative constraints of total demand \( \Delta \) and total supply \( \Sigma \). The demand and supply constraints could define the normalization factors \( \beta_{ab} \) (11)) and \( \beta_c \) (12) ) to limit the flows. When the demand of flow turn (\( \Delta_{abc} \)) exceeds the maximum capacity of the lane \( Y_{ab} \), demand of intersection outflow (\( \Delta_{c} \)) exceeds maximum capacity of intersection outflow (\( \Sigma_c \)), are calculated the normalization factor for lane or for intersection outflow, respectively.

Following is described the simplified node algorithm, developed in Matlab, divided in three parts: calculation of normalization factor for lane (Alg. 1); calculation of intersection outflow with a normalization factor for lane and for downstream link (Alg. 2); calculation of intersection inflow (Alg. 3).

The term aux refers to auxiliary variables to the algorithm and \( \Delta_{abc} \) represents the number of vehicles of the movement \( abc \) from lane \( b \) that requires to cross intersection.

**Algorithm 1** Calculation of normalization factor for lane

1) for all upstreamLink \( (a) \) do
   2) for all lane \( (ab) \) do
      3) if \#turn \( (abc) \in lane (ab) \) > 1 then
          4) for all turn \( (abc) \in downstreamLink \( (c) \) do
              5) auxy_{abc} = min \{ \Delta_{abc}, Y_{ab} \}
              6) auxy_{ab} = auxy_{abc} + auxy_{abc}
              7) end for
          8) if auxy_{ab} > \Sigma_{ab} then
              9) compute \( \beta_{ab} \)
          10) end if
      11) end if
   12) end for
13) end for

**Algorithm 2** calculation of outflow with a normalization factor for lane and for downstream link

1) for all upstreamLink \( (a) \) do
   2) Outflow = false
   3) \( \beta_c = 1 \)
   4) while Outflow = false do
      5) \( y_c = 0 \)
      6) for all turn \( (abc) \in downstreamLink \( (c) \) do
          7) \( y_{abc} = \beta_{ab} \times \min \{ \Delta_{abc}, Y_{ab} \} \)
          8) \( y_{abc} = \beta_c \times y_{abc} \)
          9) \( y_c = y_c + y_{abc} \)
      10) end for
   11) if \( y_c \leq \min \{ \Delta_c, \Sigma_c \} \) then
      12) Outflow = true
      13) else
          14) compute \( \beta_c \)
          15) end if
   16) end while
17) end for
Algorithm 3 calculation of intersection inflow

1: for all upstreamLink(a) do
2:   for all turn(abc) ∈ lane(ab) do
3:     \( y_{ab} = y_{ab}^0 + y_{abc} \)
4:   end for
5: for all lane(ab) ∈ upstreamLink(a) do
6:     \( y_a^0 = y_a^0 + y_{ab} \)
7: end for
8: end for

Where, the normalization factor \( \beta_{ab} \) is calculated as

\[ \beta_{ab} = \Delta_{abc} \sum_{c \in ab} y_{abc}^{N+1}(k) \] (11)

The normalization factor \( \beta_c \) is computed as the capacity of first cell of outgoing link divided the total flow demand direct to the outgoing link.

\[ \beta_c = \min \left\{ \frac{v_c^e(z_1(k))}{\sum_{c \in C_a} y_{abc}^{N+1}(k)} \right\} \] (12)

This general algorithms can be useful to support the transportation researchers to develop node macroscopic models with microscopic advantages in order to solve open issues on optimization of urban traffic.

IV. EXTENSIONS FOR SM

A. Database of solutions

To evaluate the gradient estimation the SM needs to calculate \( n+1 \) times the value of the objective function (see step 4), and this estimation is required many time for one solution. For this reason to increase the efficiency of SM we have memorized every solution with relative objective function (OF). If the SM requires the OF for one solution, the software avoids the run of simulation model if the OF has already been calculated.

B. Dynamic gradient step estimation

It is possible that different solutions give the same value for OF, consequently the corresponding component of the gradient, given by the difference of these solutions, is equal to 0. When the component is blocked and doesn’t change toward a better solution, we have defined a formulation to randomly increase the component of gradient equal to 0 considering the SM iterations for which the component of gradient remains equal to 0. To counts these iterations we used the counter \( z \).

\[ \eta = 0.5/k \] (13)

Moreover, respect to classic step size definition where \( k \) represents the number of SM iteration, we used the value \( u \) that counts the number of SM iteration and it becomes zero when a better solution is founded.

The calculation of the dynamic step size is described in the following algorithm.

Algorithm 4 Dynamic gradient step estimation

1: if \( \text{OF}(p^k) \leq \min \text{OF}(p^{k-1}) \) then
2:   \( u = 1 \)
3: else
4:   \( u = u + 1 \)
5: end if
6: for \( i = 1 : n \) do
7:   if \( \nabla_i L < \epsilon \) and \( \nabla_i L > \epsilon \) then
8:     \( z = z + 1 \)
9:     \( \nabla_i L = \text{rand} \in [-z, z] \)
10:    \( \eta_i = 0.5 \)
11:   else
12:     \( \eta_i = 0.5/u \)
13: end if
14: end for

C. Green splits constraints

In the optimization signal setting problem, the decision variables have to respect the constraints of green time limitation and cycle. Given an intersection with fixed-time cycle and more of two alternative phases, when the optimization method explores the possible solutions, the sum of green splits have to be equal to time cycle. To respect this constraint we proposed the following formulation

\[ \sum_{p \in \Pi_m(p)} r_m(p) = C_m , \forall m \in M \] (14)

where, \( M \) is the set of intersection, \( \Pi_m(p) \) the set of phase \( p \) indices of intersection \( m \), \( r(p) \) the green split of phase \( p \) belong to intersection \( m \). Defining with \( \psi_m \) the decision vector of the green times \( r(p) \) of the phases for intersection link \( m \) the set of neighboring feasible states, in accord to the cycle time, is defined as

\[ P(\psi_m) = \left\{ \psi_m + \tilde{r}, \forall \tilde{r} \in 0,1 \Pi_m(p) \mid \sum_{p \in \Pi_m(p)} r_m(p) = C_m \right\} \] (15)

For each phases of one intersection only one green time is defined as decision variable. If the green time changes, the neighboring feasible states changes in accord to the cycle time. For example, for one intersection with cycle time of 100 sec and three alternative phases, given the green times [50, 25, 25] sec the decision vector considers only the first element. If this element is changed of +1 sec, the set of neighboring feasible states corresponds to ([51,24,25], [51,25,24]). The complete decision vector is evaluated with either solutions and the green times are fixed for intersection that returns the best OF value.
V. RESULTS

The following results show the effects in terms of traffic predictions and efficiency of the CMT-UT compared to the macroscopic model. The analysis is based on the following microscopic elements are used:

- queues dissipation in the channelized lanes at upstream intersection;
- conflict flows that could occur when the vehicles try to enter in channelized lanes;
- representation of different turn movements belonging at the same lane.

A. Effects of microscopic elements on traffic prediction

We have defined a hypothetical test network with 9 four-way signal controlled intersection and 12 centroid for the flow of input/output of network. All links (300 meters in length) have max capacity \( Y \) equal to 900 [veic/hour] and two lanes. The capacity and demand of the links are split over the two lanes fifty-fifty, excepted for the principal paths where the demands are represented in the Fig.1.

\[
TD = \sum_{h=1}^{H} D(h) \tag{17}
\]

The CTM-UT represents all links with a channelized zone of 120 meters, instead the macroscopic model does not represent channelized zone and vehicular conflict at the link. In this macroscopic representation the traffic light, and relative spillback phenomena that causes delay, effecting the total flow only at the end of the link. In case of urban channelized zone, the CTM-UT allows to obtain a correct representation of the urban dynamics with a loss of efficiency around 4% respect to classical CTM. Such loss of efficiency is due to the additional representation of the propagation into the channelized zone of the link.

The CTM-UT can represent behaviors of urban drivers, its peculiarity improves the efficacy of travel prediction for complex urban intersections. It is possible considered an additional movement belonging to the right-turn so that the demand lane is split in right-turn (95% turn demand) and through (5% turn demand) movements. This through demand is applied to all lanes for right-turn of network and it belong to the phase 1 for east-west/west-east directions and to phase 2 for south-north/north-south. These percent represent drivers that must move forward but for error take the channelized lane for right-turn.

Such analysis highlighted that to obtain efficacy solutions for urban traffic optimization, it is necessary to model flows that crosses the channelized lanes, flow upstream intersection and the complex flow intersection with detail level quite close to the microscopic models.

B. Surrogate Model with new extensions

In this section are presented the results of the new SM extensions applied to the test network in Fig. 1 with the inflow demands of 800 [veic/h] from the nodes 11,13,15,18 during a simulation time of 6,66 min.
Four versions of the optimization method based on CTM-UT are defined: classic SM with step size presented in (13); SM with Database of Solutions (SM-DS); SM with Dynamic Gradient step estimation (SM-DG) and SM both with DS and DG (SM-DSDG). Each version of SM are compared in terms of efficacy (value of $TD$) and efficiency (number of times that the OF must be calculated $NOF$). The initial state is defined with the total cycle time over the two phases fifty-fifty, $50$ sec for each decision variables. The optimization model is stopped after $29$ iterations ($k$).

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VI. CONCLUSION

These preliminarily experiments put in evidence that CTM-UT can be well-applied for the real-time signal setting problem. It is suitable to represent the accuracy of microscopic elements with an increase of computational time around 9% respect to classic macroscopic model. To obtain effective optimization solutions for urban network it is important that the simulation model represents with accuracy the flows upstream intersection.

The results obtained with the proposed SM extensions demonstrated that the new dynamic gradient step estimation improves the efficacy about 10% respect to the classic step estimation; and when the Database of Solutions extension is applied the improvement is 4%. The new dynamic gradient step estimation permits to avoid the local optimum and it increases the number of times that the objective function is calculated.

The new SM based on CTM-UT characteristics has the potential to settle some unresolved critical issues on the real-time applicability of the ITS for urban traffic networks on large-scale.

REFERENCES


