Abstract—In this paper, modified model-following sliding mode control based on the active disturbance rejection control observer is proposed and demonstrated. We introduce the Active disturbance rejection control with disturbance observer. the Improved ADRC is normalizing the plant and the removal characteristics of the input-side disturbance. Therefore, the plant is changed to the characteristics not having the plant parameters. The controller is used the sliding mode control. In the sliding mode control, the reachability condition is improved the adaptive control by using the improved ADRC. The simulation studies are applied to a plant with stable, unstable terms, and a stable plant with an integrator. Simulation results show that this method can get the superior performance of positioning control.

Key Words—Sliding mode control (SMC), Active disturbance rejection control (ADRC), Reachability condition

I. INTRODUCTION

The control of the positioning of a robotics posture and/or joints has become increasingly important. Proportional (P), Proportional-Integral (PI), and/or Proportional-Integral-Derivative (PID) control are the most common techniques in classical control theory used for positioning control. However, PID control is not an optimal approach and new methods have evolved for positioning control, including model-following control.

The model-following control system [1]-[5] has been studied as a classical design technique by various researchers. The purpose of a model-following system is to make the plant output follow the output of a reference model that satisfies the reachability condition. Therefore, we focus on the active disturbance rejection control (ADRC) [6]-[7]. ADRC is a design method used for uncertainty system such as internal disturbances, unmodeled dynamics, and external disturbances are one of the states of system. ADRC can be used as the control signal to compensate for real perturbations in the plant and can provide feedback to cancel the uncertainties.

From this, we designed to expand ADRC including the disturbance, the improved ADRC.

As the controller designing, SMC [8]-[11] is effective method and a variable structure control method. One of the features of SMC is its excellent robustness to disturbances and it also has the ability to model system uncertainties that satisfy the reachability condition. For example, Wang et al. [12] method is the effective model-following sliding mode control method. However, the reachability condition is not satisfied, plant output gets the influence of the disturbance. Therefore, we design the modified model-following sliding mode control method by using the improved ADRC. The modified sliding mode controller is designing based on Wang et al. [12] method.

This paper is organized as follows: The first section is describes "Description of plant structure", "The improved Active disturbance rejection control" describes to normalize the plant. "The description of model structure" shows the designing of model. "Modified model - following sliding mode control" propose the design of SMC controller and improving the reachability condition. "Simulation study", the simulation study show the effectiveness of our proposed method in several situations. Finally, we present our conclusions.

II. THE DESCRIPTION OF PLANT STRUCTURE

This section describes the plant and model structure. The generalized plant and model in the positioning of the robotics posture and the motor are described as follows:

\[
G_p(s) = \frac{Y_p(s)}{U_p(s)} = \frac{b_p}{s^{n-1} + \sum_{i=2}^{p} a_{pi} s^{i-2}}. \tag{1}
\]

where the plant parameters are defined as \(a_{pi}\) and \(b_p\). \(U_p(s)\) and \(Y_p(s)\) are Laplace transforms of the manipulated variable \(u_p(t)\), and the plant output \(y_p(t)\), respectively. In this paper, let the transfer function of the third-order plant be

\[
G_p(s) = \frac{b_p}{s^3 + a_{p2}s^2 + a_{p1}s + a_{p0}}. \tag{2}
\]
These transfer functions are expressed in the form of the following state-space equations.

\[
\begin{align*}
\dot{x}_p(t) &= A_p x_p(t) + b_p u_p(t), \\
y_p(t) &= c_p x_p(t),
\end{align*}
\]

(3)

where

\[
x_p(t) = \begin{bmatrix} x_{p1}(t) \\
x_{p2}(t) \\
x_{p3}(t) \end{bmatrix}, \quad A_p = \begin{bmatrix} 0 & 1 & 0 \\
0 & 0 & 1 \\
a_{p1} & a_{p2} & a_{p3} \end{bmatrix},
\]

\[
b_p = \begin{bmatrix} 0 \\
0 \\
b_{p} \end{bmatrix}, \quad c_p = [1 \ 0 \ 0].
\]

\(A_p, b_p,\) and \(c_p\) are the constant matrices of appropriate dimension for the plant, respectively. \(x_p(t)\), and \(\dot{x}_p(t)\) are the plant state vector, and a differential of the plant state vector, respectively.

III. THE IMPROVED ACTIVE DISTURBANCE REJECTION CONTROL

This section describes the improved active disturbance rejection control for normalizing the plant and the removal characteristics of the input-side disturbance. The plant is defined Eqs. (2) and (3). The construction of Eq. (3) has been written as following equation.

\[
\begin{align*}
\dot{x}_{p1}(t) &= x_{p2}(t) \\
\dot{x}_{p2}(t) &= x_{p3}(t) \\
\dot{x}_{p3}(t) &= -a_{p1}x_{p1}(t) - a_{p2}x_{p2}(t) - a_{p3}x_{p3}(t) + b_p u_p(t)
\end{align*}
\]

(4)

Including the input-side disturbance \(d(t)\), \(\dot{x}_{p3}(t)\) can be written as follows:

\[
\begin{align*}
u_p(t) &\Rightarrow u_p(t) + d(t), \\
-a_{p1}x_{p1}(t) - a_{p2}x_{p2}(t) - a_{p3}x_{p3}(t) &\Rightarrow f(t), \\
b_p u_p(t) &\Rightarrow \ddot{u}(t), \\
b_p d(t) &\Rightarrow \dot{d}(t).
\end{align*}
\]

(5)

\[
\begin{align*}
\dot{x}_{p3}(t) &= f(t) + b_p u_p(t) + b_p d(t) \\
&= f(t) + \ddot{u}(t) + \dot{d}(t)
\end{align*}
\]

(6)

Assuming the input-side disturbance \(\dot{d}(t) = 0\) and to eliminate the influence of the disturbance, \(\dot{x}_4(t)\) is rewritten as follows:

\[
\dot{x}_4(t) = h \ddot{f}(t) = h \dot{d}(t)
\]

(8)

\(h\) is an arbitrary constant. Eq. (10) is rewritten as follows:

\[
\begin{align*}
\dot{x}(t) &= A x(t) + b \ddot{u}(t) + b \ddot{f}(t), \\
y(t) &= c x(t)
\end{align*}
\]

(9)

where,

\[
x(t) = \begin{bmatrix} x_1(t) \\
x_2(t) \\
x_3(t) \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & h \end{bmatrix},
\]

\[
b_1 = \begin{bmatrix} 1 \\
0 \\
0 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 0 \\
0 \\
1 \end{bmatrix}, \quad c = [1 \ 0 \ 0 \ 0].
\]

The observer is written based in the improved ADRC.

\[
\begin{align*}
\dot{\hat{x}}(t) &= A_o \hat{x}(t) + b_o \ddot{u}(t) - L_o \left( y_p(t) - \hat{y}(t) \right), \\
\hat{y}(t) &= c_o \hat{x}(t)
\end{align*}
\]

(10)

where,
IV. THE DESCRIPTION OF MODEL STRUCTURE

This section describes model structure. The generalized model is described as follows:

\[
G_m(s) = \frac{Y_m(s)}{U_m(s)} = \frac{b_m}{s^{n-1} + \sum_{i=2}^{n} a_{m(i-1)} s^{i-2}}.
\]  

(13)

where the model parameters are defined as \(a_{mi} \) and \(b_m \) where \(b_m = a_{m1} \). \(U_m(s)\), and \(Y_m(s)\) are Laplace transforms of the model input \(u_m(t)\) = target-value \(r(t)\), and the model output \(y_m(t)\), respectively. In this paper, let the transfer function of the third-order model be

\[
G_m(s) = \frac{b_m}{s^3 + a_{m2}s^2 + a_{m2}s + a_{m1}}.
\]  

(14)

These transfer functions are expressed in the form of the following state-space equations.

\[
\begin{align*}
\dot{x}_m(t) &= A_m x_m(t) + b_m u_m(t), \\
y_m(t) &= c_m x_m(t)
\end{align*}
\]  

(15)

where

\[A_m, \quad b_m, \quad c_m\]

\(A_m, b_m, \) and \(c_m\) are the constant matrices of appropriate dimension for the model, respectively. \(x_m(t), \dot{x}_m(t)\) are the plant state vector, a differential of the plant state vector, the model state vector and a differential of the model state vector, respectively.

The error dynamics of the normalized plant and the model are given as follows:

\[
\begin{align*}
\dot{x}_{np}(t) &= A_{np} x_{np}(t) + b_{np} u_{np}(t), \\
y_{np}(t) &= c_{np} x_{np}(t)
\end{align*}
\]  

(12)

where

\[
\begin{align*}
x_{np}(t) &= \begin{bmatrix} x_{np1}(t) \\ x_{np2}(t) \\ x_{np3}(t) \end{bmatrix}, \\
A_{np} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \\
b_{np} &= \begin{bmatrix} 0 \\ b_{np} \\ 1 \end{bmatrix}, \\
c_{np} &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.
\end{align*}
\]  

The error dynamics of the model is described as follows:

\[
\dot{e}(t) = A_i e(t) + \left( A_{np} - A_m \right) x_{np}(t) + b_{np} u_{np}(t) - b_m r(t).
\]  

(17)
Eq. (17) is satisfied the following matching condition.

\[
\begin{align*}
A_m - A_{np} &= b_{np} K_1, \\
b_m &= b_{np} K_2.
\end{align*}
\]  

(18)

\(K_1\) and \(K_2\) are obtained as following equation.

\[
\begin{align*}
K_1 &= b_{np}^+ (A_m - A_{np}) \\
K_2 &= - (c_m A_m b_{np})^{-1}
\end{align*}
\]  

(19)

where, \(b_{np}^+\) is the Moore-Penrose Matrix Inverse of \(b_{np}\). \(K_2\) is defined to satisfy the target-value.

V. MODIFIED MODEL - FOLLOWING SLIDING MODE CONTROL

This section proposes the modified model - following sliding mode control. The modified sliding mode controller is designed by using the improved ADRC. First, the argument system is defined as follows:

\[
\dot{e}_s (t) = A_s e_s (t) + b_s u_{np} (t)
\]  

(20)

where,

\[
e_s (t) = \begin{bmatrix} e(t) \\ z(t) \end{bmatrix}, \quad A_s = \begin{bmatrix} A_m & 0 \\ e_m & 0 \end{bmatrix}, \quad b_s = \begin{bmatrix} b_{np} \\ 0 \end{bmatrix},
\]

\[
\dot{z}(t) = y(t) - y_m (t).
\]

The switching function \(\sigma(t)\) is determined for the error dynamics as follows:

\[
\sigma(t) = S e_s (t)
\]  

(21)

\(S\) is the switching surface. The equivalent linear input is generally obtained through \(\dot{\sigma}(t) = 0\) so that the switching function converges to 0, \(\sigma(t) = 0\). \(\dot{\sigma}(t)\) is written as follows:

\[
\dot{\sigma}(t) = S \dot{e}_s (t)
\]  

(22)

\[
= S \left\{ A_s e_s (t) + b_s u_{np} (t) \right\}
\]

\[
= S A_s e_s (t) - S b_s \left\{ K_s x_{np} (t) + K_2 r (t) - u_{np} (t) \right\}
\]

(23)

\(u_{np} (t)\) is the equivalent linear input \(u_{in} (t)\) is following equation.

\[
u_{in} (t) = -(S b_s)^{-1} S A_s e_s (t) + K_1 x_{np} (t) + K_2 r (t)
\]

To obtain the switching surface \(S\), we introduce the following cost function.

\[
J = \int_0^\infty \left\{ e_s^T (t) Q e_s (t) + u_{in}^T (t) R u_{in} (t) \right\} dt,
\]  

(24)
where $Q$ is the weight matrix, $R$ is the weight coefficient, $R=1$. The switching surface $S$ that minimizes the cost function is then given by

$$S = \tilde{B}^T P.$$  \hspace{1cm} (25)

$P$ is the solution of the matrix Riccati equation.

$$PA_s + A_s^T P - Pb_s Rb_s^T P + Q = 0.$$  \hspace{1cm} (26)

Sliding mode control includes the linear term and the non-linear term. A non-linear input is determined as follows:

$$u_{nl}(t) = -k \text{sgn}\{\sigma(t)\} = -k(SB_s)^{-1}\frac{\sigma(t)}{[\sigma(t)]^T + \delta}$$  \hspace{1cm} (27)

where, $k$ is the switching gain, $\sigma(t)$ is the switching function, and $\delta$ is the positive coefficient for the suppression of chattering. Here, when the input-side disturbance is inserted to the system, the reachability condition of the sliding mode is as follows:

$$\dot{V} = \sigma(t)\dot{\sigma}(t) = \sigma(t)[SA_s e_s(t) - SB_s[K_1x(t) + K_2r(t) - u_{nl}(t) + d(t)]]$$  \hspace{1cm} (28)

Eq.(1.2) is calculated as follows:

$$\dot{V} = \sigma(t)\dot{\sigma}(t)$$
$$= \sigma(t)[SA_s e_s(t) - SB_s[K_1x(t) + K_2r(t) - u_{nl}(t) + d(t)]]$$
$$= -SB_s[k\frac{\sigma(t)^2}{[\sigma(t)]^2 + \delta} + \sigma(t)d(t)]$$
$$= -SB_s[k\sigma(t) + \sigma(t)d(t)]$$  \hspace{1cm} (29)

In Eq. (29), because $SB_s > 0$, the reachability condition is satisfied by the following equation.

$$k > |d(t)|$$  \hspace{1cm} (30)

The maximum estimated value of the disturbance is defined as $d_{\text{max}}(t)$. $k$ is determined to satisfy the following equation.

$$k > |d_{\text{max}}(t)|$$  \hspace{1cm} (31)

In general, the switching gain $k$ is determined in advance. However, if $k$ does not satisfy the reachability condition, plant output gets the influence of the disturbance. Therefore, $k$ is designed as the adaptive gain by using the improved ADRC in the proposed method.

VI. SIMULATION STUDY

In this section, the effectiveness of the proposed method is evaluated in the simulation studies.

A. Plant and Model settings

The evaluated plants and the model are described as follows:

Plants

1) Stable Plant [12]

$$G_p1(s) = \frac{2.2}{s^3 + 5.2s^2 + 8.7s + 4.4}$$  \hspace{1cm} (32)

2) Unstable Plant

$$G_p2(s) = \frac{2.2}{s^3 - 5.2s^2 + 8.7s + 4.4}$$  \hspace{1cm} (33)

3) Stable plant with an integrator

$$G_p3(s) = \frac{2.2}{s(s^2 + 5.2s + 8.7)}$$  \hspace{1cm} (34)

Model

$$G_m(s) = \frac{5}{s^3 + 3s^2 + 6s + 5}$$  \hspace{1cm} (35)

Here, the target-value $r(t)$ is set to 1.0 and the input-side disturbance $d(t)$ of magnitude -200 is inserted at $t = 15$ sec.

B. The improved ADRC and SMC parameters

The improved ADRC parameters are described as follows:

$$h = 100,$$
$$Q = \text{diag}[10^5 \ 10^5 \ 10^5 \ 10^{20} \ 10^{25}],$$
$$R = 1,$$
$$L_0(t) = \begin{bmatrix}
3.61 \times 10^3 \\
6.45 \times 10^6 \\
6.82 \times 10^9 \\
6.21 \times 10^{11} \\
3.16 \times 10^{12}
\end{bmatrix}$$  \hspace{1cm} (36)
SMC parameters are as follows:

\[
\begin{align*}
\delta &= 0.1, \\
Q &= \text{diag}[1 \times 10^2 \ 1 \ 1], \\
S &= [7.08 \ 4.51 \ 1.36 \ 1].
\end{align*}
\]

VII. CONCLUSION

We have proposed modified model-following sliding mode control based on the active disturbance rejection control. The improved ADRC is effective control method to normalize the plant and to remove the input-side disturbance. Based on the improved ADRC, the switching gain is designed by the adaptive control using the improved ADRC for improving the reachability condition. The simulation studies get the superior performance of positioning control in the several cases.

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