Lateral Stability Control Based on Active Motor Torque Control for Electric and Hybrid Vehicles

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Abstract - This paper describes a method for lateral stability control of electric and hybrid vehicles utilizing multiple electric motors connected to independent wheels of the vehicle. Independent motor torque control based on LQR and PID are used to generate positive drive and negative brake torques for imposing an aligning moment around the yaw axis resulting in the lateral stability functionality. The developed algorithms were implemented and performance tested in Matlab-Simulink environment and the dynamic performance characteristics were reported with numerical simulation results.

Keywords - yaw control, lateral stability, PID, LQR, hybrid vehicle

I. INTRODUCTION

Nowadays, especially concerning traffic safety, handling of motor vehicles is one of the most important subjects. Since safety and comfort is essential vehicle stability control has been an highly important concept. Especially as hybrid technology become more common, active control systems which consider hybrid design is studied more. There are great numbers of study in literature about roll dynamics control via torque control for in-wheel motors\[1,2,3\]. In reference \[4\], H-infinity based state feedback controllers are designed which is resistant to tyres working condition in linear and nonlinear regions. Another stability conrol study proposed by Kim, J. and Kim, H. is yaw control algorithm for in-wheel motor vehicle\[5\]. Among vehicle stability control strategies fuzzy logic based control is also widely used\[6,7,8\]. Limited availability of necessary sensors for feedback signals to designing control system is also caused the fuzzy technique to be used more widely\[9\]. The authors of \[10\] utilize yaw moment control and recommend ITB (Independent Axle Torque Biasing) in order the vehicle handling to be close to a linear vehicle handling characteristic in normal driving situations and to maintain the vehicle dynamics within the stable handling region in extreme maneuvers. In literature there are various methods and models used for analyzing and handling vehicle stability, single track model is generally used to detect factors which affect stability. In our study, single track model is used as reference model. We will represent how yaw stability affected using single track model with roll degree of freedom using electric motors at rear wheels.

II. VEHICLE MODELING

For the beginning, linear bicycle model (single track model) which is used commonly in literature, considered sufficient to observe and detect the main dynamics of the vehicle. In order to get more realistic results roll degree of freedom is included in model.

A. Single Track Model

The Linear bicycle model (Figure 1), is widespread used to explain the dynamic behaviours and lateral/longitudinal responses of road vehicle, under the following assumptions:

- Motions of body roll, pitch and bounce are neglected
- The vehicle is assumed to has constant velocity
- Only external force acting on the vehicle is the lateral tire Force which is proportional to slip angle and occurs under assumption of low slip angle

Figure 1. Linear bicycle model
where:

\[ \begin{align*}
\alpha & \text{ distance from front axle to centre of gravity} \\
\beta & \text{ distance from rear axle to centre of gravity} \\
C_{af} & \text{ cornering stiffness of front axle} \\
C_{ar} & \text{ cornering stiffness of rear axle} \\
I_z & \text{ yaw moment of inertia} \\
m & \text{ vehicle mass} \\
\Psi & \text{ vehicle yaw rate} \\
U & \text{ absolute vehicle velocity} \\
\beta & \text{ vehicle slip angle} \\
\delta_f & \text{ steer angle of front wheels} \\
\alpha_f & \text{ front wheel slip angle} \\
\alpha_r & \text{ rear wheel slip angle} \\
F_{yf} = C_{af} \alpha_f, \quad F_{yr} = C_{ar} \alpha_r
\end{align*} \] (1)

- The right and left wheel characteristics are combined in an equivalent wheel characteristic. Axle lateral wheel forces are defined by linear tire model as a function of slip angle.

Considering the slip angles of vehicle \( \beta \) to be small, the vehicle lateral acceleration and slip angles of both axles can be expressed [11] as:

\[ \begin{align*}
a_y & = U(\beta + \Psi) \\
\alpha_f & = \delta_f - \beta - a_y \\
\alpha_r & = -\beta + b_y U \\
\Psi & = \frac{b_{car} - a_{car}}{I_z} \beta - \frac{a_{car}^2 + b_{car} a_{car}}{I_z U} \Psi + a_{car} \delta_f \\
\beta & = \frac{- C_{af} a_{car}}{mU} \beta + \left( 1 - \frac{a_{car} a_{car}}{mU^2} \right) \Psi + \frac{a_{car}}{mU} \delta_f
\end{align*} \] (2-6)

In order to describe the dynamic behaviour of the bicycle model, a state space model will be used, which can easily be derived from the equations of motion with state \( x \), input \( u \) and output \( y \). The state space model can be written in the following way:

\[ \begin{align*}
\dot{x} & = Ax + Bu \\
A & = \begin{bmatrix}
\frac{C_{af} + C_{ar}}{mU^2} & \frac{b_{car} - a_{car}}{mU} & \frac{a_{car}^2 + b_{car} a_{car}}{mU^2} & \frac{a_{car}}{mU} \\
\frac{b_{car} - a_{car}}{mU} & \frac{1}{I_z} & \frac{a_{car}^2 + b_{car} a_{car}}{mU I_z} & \frac{a_{car}}{mU I_z} \\
\frac{a_{car}^2 + b_{car} a_{car}}{mU^2} & \frac{a_{car}^2 + b_{car} a_{car}}{mU^2} & \frac{1}{I_z} & \frac{a_{car}}{mU I_z} \\
\frac{a_{car}}{mU} & \frac{a_{car}}{mU I_z} & \frac{a_{car}}{mU I_z} & \frac{1}{I_z}
\end{bmatrix} \quad B = \begin{bmatrix}
\frac{C_{af}}{mU} \\
\frac{C_{ar}}{mU}
\end{bmatrix}
\end{align*} \] (7-8)

\[ x = \begin{bmatrix} \beta \\ \Psi \end{bmatrix}, \quad u = \delta_f = \frac{\delta}{l_z} \] (9)

At equation (1.9), \( l_z \) is steering ratio, \( \delta_f \) is steering angle of front wheels and \( \delta_h \) is input signal (steering angle).

\[ x = [\beta, \Psi, \phi, \varphi] \] is the state vector while input vectors are \( \delta \) (steering angle) and \( u \) represents the braking/traction force which is applied by electric motors that are mounted to rear wheels of the vehicle. This force is used to enable yaw and rollover control [12]. PID and LQR control techniques are tested on this model for tracking yaw rate.

III. CONTROLLER DESIGN

Yaw and slip motions have huge effects on vehicle handling and stability, and these features can be controlled via yaw moment. In the proposed model, rear wheels are controlled via electric motors, in case of understeering or oversteering situations, yaw moment which is generated by this electric motors will contribute to stability. Reference values are calculated for yaw rate and side-slip angle (\( \Psi \) and \( \beta \) ) and tracking this reference values is aimed using PID and LQR controllers. First model shows tracking of \( \Psi_{ref} \) with single track model using a control moment input that will be occured with braking or traction forces applied to the rear wheels by wheel motors. Assuming yaw rate is measured and sideslip angle is estimated properly, the control system is shown in the block diagram below. Same control logic is used both for PID and LQR.

![Figure 2. Control scheme](image)

The main purpose of controller is tracking the \( \Psi_{ref} \) by minimizing the difference between ideal and actual yaw rates. With properly adjusted parameters for controller, corrective moment \( M_z \) is obtained to determine the control
torque to apply on wheels. Total torque that can be obtained from this controller is given in equation (11) but since we use two electric motors on rear wheels, the control moment we can apply is the total moment on rear wheels [10] as depicted in equation (12).

\[ T_t = (F_{xfl} + F_{xfr} + F_{xrl} + F_{xrr})R \]  
(11)

\[ M_z = (F_{xrr} - F_{xrl}) \frac{T}{2} - \frac{r_{xrr} - r_{xrl}}{R} \]  
(12)

For the simulations that is shown below, desired yaw rate and desired sideslip angle is calculated as follows [13]:

\[ \psi_{des} = \frac{\delta v}{l(1 + \frac{\rho^2}{\rho_{cent}})} \] or \[ \psi_{des} = \frac{a_2}{v} \]  
(13)

\[ \beta_{des} = \frac{\rho}{\rho_{center}} \frac{m b v^2}{\epsilon_{air ip}} \]  
(14)

A. PID Controller

The control scheme given above is designed. The desired sideslip angle and desired yaw rate are calculated using reference model. Minimizing the difference between desired and observed yaw rates via controller, yaw torque \( M_z \) is generated as a corrective moment which will be the control input. In order to do this, PID controller is used due to its simple structure and common usage. PID controller is composed of 3 parameters that are proportional (\( K_p \)), integral (\( K_i \)) and derivative (\( K_d \)) control parameters. Determining these parameters, various methods can be used like Ziegler-Nichols method or tuners, in this study, we used PID toolbox in Matlab/Simulink. The difference between reference yaw rate(13) and observed yaw rate (\( \psi_{ref} - \psi \)) is used as an input continuously for controller and our control torque, \( M_z \) is obtained as an output.

\[ M_z = K_p \cdot e + K_i \int e \, dt + K_d \frac{de}{dt} \]  
(15)

Double lane change maneuver test is performed on single track model with roll degree of freedom and simulated in Matlab/Simulink. Yaw rate response and required corrective moment (\( M_z \)) and the effect of corrective yaw moment on roll rate are shown in the graphs. \( M_z \) has a small effect on roll rate, it is seen that controlled yaw rate can track the reference yaw rate with acceptable error according to yaw rate output. However, the required torques are too high to be generated by electric motors, LQR control is also applied on same model.

B. LQR Controller

LQR controller obtains feedback gain matrix by minimizing the cost function which is given below.

\[ J = \int_0^\infty x^T Q x + u^T R u \, dt \]  
(16)

The gain matrix \( K \) is obtained from solution of Ricatti equation which is given below. Controller is robust when there can be found a \( K \) matrix with positive semidefinite \( R \) and \( Q \) matrices satisfying Ricatti equation

\[ KA + A^T K - KB^{-1}B^T K + Q = 0 \]  
(17)

Being positive semidefinite matrices we identify \( R \) and \( Q \) as; \( R=1 \) and \( Q=[50000 \quad 0 \quad 0 \quad 500000] \) than controller is designed according to the equation (18). Here, “\( u \)” represents control input \( (M_z) \) and \( K \) matrix is multiplied by \( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \) where \( x_1 \) and \( x_2 \) represents side slip angle and yaw rate respectively. Seeing coefficients of \( Q \), it can be noticed yaw control is emphasized in control.

\[ u = -K x = -[K_1 \ K_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \]  
(18)

Using double lane maneuver test, LQR controller is tested on the linear bicycle model with roll degree of freedom in order to compare with performance of the PID controller. According to the test results, yaw rate is higher than PID controller and required torque is less than the required torque with PID controller although there is no big difference between the results of PID and LQR control considering linear bicycle model with roll degree of freedom. Additionally, the roll rate is also better with yaw control. Matlab/Simulink test results are given below.
IV. LATERAL STABILITY(YAW) CONTROLLED 4 WHEEL TRACK MODEL

In order to observe the dynamic characteristics of the effectiveness of in wheel motors on lateral stability(yaw), a track model is simulated using both PID and LQR control techniques. Using this model we get more realistic results and torque distribution on each wheel can be observed. For linear bicycle model total moment at rear axle is considered however in this model total moment at rear axle is distributed to left and right wheels. This full track model includes longitudinal, lateral, yaw motions and rotational dynamics of the four wheels. Pitch and vertical motions are neglected in this model. Considered dynamics are given below[14]:

Longitudinal movement:

\[ m(V_x - V_y \dot{\psi}) = (F_{xf} + F_{xf})\cos \delta - (F_{yf} + F_{yf})\sin \delta + F_{xrr} \mp F_{xrl} \]

Lateral movement:

\[ m(V_x - V_y \dot{\psi}) = (F_{yf} + F_{yf})\cos \delta + (F_{xf} + F_{xf})\sin \delta + F_{yrr} \mp F_{yrl} \]

Yaw movement:

\[ I_x \dot{\psi} = l_o (F_{yf} + F_{yf})\cos \delta - l_o (F_{xf} + F_{xf})\sin \delta - I_h (F_{yf} + F_{yf})\sin \delta - I_h (F_{xf} + F_{xf})\sin \delta + \frac{d}{2} (F_{xrr} - F_{xrl}) \]

Since the forces which to be controlled are provided by electric motors that are mounted to rear wheels, longitudinal force occurs at rear wheels. Thus, the longitudinal force on front wheels are the same \( F_{xf} = F_{xf} \). Considering full track vehicle model, since \( \delta \) is a small angle, \( \sin \delta \) is neglected. In the equations below; \( M_z \) represents the corrective moment which is allocated to left and right rear wheels. \( R \) and \( d \) represents wheel radius and d the wheelbase. In this case;

\[ I_x \dot{\psi} = l_o (F_{yf} + F_{yf})\cos \delta - I_h (F_{yf} + F_{yf})\sin \delta + \frac{d}{2} (F_{xrr} - F_{xrl}) \]

\[ M_z = \frac{d}{2} (F_{xrr} - F_{xrl}) \]

(22)

The left and right braking(or driving) torques \((T_{xrl}, T_{xrr})\) are obtained as below; [16]

\[ T_{xrl, xrr} = \frac{M_z}{d} \]

(24)

According to the model, when the vehicle loses stability, one of the rear wheel motors will apply brake or driving force, this motors changes the yaw moment on the vehicle via mentioned forces in order to improve the stability. Distribution of the yaw torque that occurs at rear part of the vehicle is basically rely on increasing driving or breaking force on another wheel simultaneously. From the equations above, we can derive the left and right wheel torques dependently as follows;

\[ T_{xrl} = \frac{M_z}{d} R \]

(25)

In above, \( T_{xrl} \) and \( T_{xrr} \) are left and right driving(or braking) torques are calculated however, these torques are limited by the road friction conditions. Deciding if the force that will be applied to the wheel is driving force or braking force or determining which wheel this force will be applied to depend on steering wheel angle and the sign of the yaw error (understeering/oversteering situations). For example, if the vehicle is turning right and actual yaw rate is less than desired yaw rate which means the vehicle is understeering, the brake is applied to the right rear wheel. For this full vehicle model, corrective moment \( M_z \) is calculated by both PID and LQR controllers, and simulated in Matlab/Simulink results are given below.

A. PID Controller

PID controller is applied to full track model to track the desired yaw rate and double lane maneuver test is conducted on model. Steering wheel angle input, yaw rate response and required torque from rear in wheel motors are given in graphs. According to results, this controller does not have good performance on our model, required torque is too high that electric motors are not able to satisfy that value.

B. LQR Controller

When full track model is tested, it can be seen that LQR controller is more effective than PID controller for yaw rate control. Without high torques, acceptable yaw rates can be obtained under control. Compared to LQR, PID control has results which are not satisfactory with high torque need. Till now, controllers generally required excessive torques that we couldn’t foresee. These high torques are not easy to obtain from electric motors in order to provide desirable yaw rates. For a solution to this problem we applied brake to front wheels, this implementation will help to achieve yaw control with smaller torques from electric motors. The results of double lane maneuver test with LQR controller are given below:
As explained above, a control algorithm is developed for front brakes to step in and Load Transfer Ratio ($\text{LTR}_d$) is used as input signal to controller. This ratio states the vehicle’s roll behaviour and expressed as following:

$$\text{LTR}_d = \frac{z}{mgT} (k\phi + c\phi)$$  \hspace{1cm} (26)

Deciding when to activate brakes, ($\text{LTR}_d$) and the rate of change of dynamic load transfer ratio ($\text{LTR}_d$) are taken into account. According to first condition, when $\text{LTR}_d$ exceed a threshold value (0.75) brakes are activated. For the system to be deactivated again $\text{LTR}_d$ must reach to lower threshold value (0.65) for more stable performance.

However, in case of some abrupt maneuvers, when $\text{LTR}_d$ increase too fast, early intervention might be needed. In this situations, $\text{LTR}_d$ is used as decision parameter. According to the second condition relay is arranged to be open when the value of $\text{LTR}_d$ exceed 0.95 ; and closed when it is below 0.80. When at least one of these relays is open, roll over prevention is active with front brakes.

The situation with active front brakes and rear electric motors is simulated using a commercial vehicle simulator.

V. ANALYSIS AND RESULTS

The controllers which mentioned under IV. Title applied on the linear model which mentioned in III. and acceptable results has observe. After that implementation our control strategies which are effective on linear model then used with more complex 4 wheel vehicle model and still able to generate reasonable results. In full vehicle model performance which simulated in commercial vehicle simulator, we get more realistic results due to torque distribution and further dynamics of vehicle(tire dynamics etc.) that are included unlike linear model.

Mentioned simulator uses real vehicle model that includes all vehicle dynamics. According to the scenario, the vehicle cruise at 120 km/h, undergoes double lane change maneuver test using electric motors and front brakes. When front brakes are active stability could be improved by lower torques from electric motors. Simulation results are given below.
Figure 9. Braking Torques

Seeing simulation results for yaw control, maximum torque can be obtained from electric motor is limited with 100 Newtons each. However this torque is not enough to provide stability without front brakes, with the brake force shown above, animations show that, the vehicle could track the desired trajectory closely.

ACKNOWLEDGMENTS

This work was funded by TUBITAK grant number 113M070 and was facilitated by Gediz University Mechanical Engineering Department and Hacettepe University Automotive Engineering Department. The authors wish to thank the funding organization and the host institutions for their support.

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